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A STUDY OF HIGH ORDER BORN APPROXIMATIONS FOR LOW GRAZING ANGLE HIGH FREQUENCY ROUGH INTERFACE SCATTERING

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ABSTRACT

The first Born approximation is an effective method for the prediction of scattering from rough interfaces. This calculation arises from retaining only the first term in the Neumann series of the solution of an inhomogeneous integral equation of the second kind. This method yields adequate results for some geometries and frequencies. However, it cannot account for the multiple scattering which is expected to occur at low grazing angles for sufficiently rough and appropriately spaced rough surfaces. The reason for this is that multiple scattering is not included in the mathematics of the first Born approximation. However, second and higher order terms do allow for secondary, tertiary, etc., scattering and should prove to be a good predictive tool for low grazing angles for some geometries and all grazing angles for other cases. In the study presented here, we develop a technique for calculating higher order Born terms iteratively by a numerically efficient method. The formulation is then used to study the effects of scattering from rough interfaces for a variety of interesting cases for the first two Born terms.

INTRODUCTION

The rigorous mathematical study of acoustic scattering from rough surfaces dates as far back as J. W. S. Rayleigh [1]. Much of the early work was concerned with the description of the acoustic field after a primary (single) scattering event from a slightly rough surface. An iterative procedure whereby a reasonable first guess for the pressure on the surface is used to calculate the scattered field which is in turn used as the second guess to calculate an improved scattered field, etc., is a well-known [2] lengthy process that converges to the correct answer. More recently, researchers have been concentrating on the more difficult

problem of multiple scattering from rough surfaces [3,4,5]. In this paper we expand on our earlier studies [6,7] of using higher order Born (also known as Neumann) approximations to account for sequential scattering from rough surfaces. By judicious formulation, as opposed to brute force techniques, this sequential procedure can be used efficiently to account for secondary scattering. In Section 1, the theoretical development of the higher order Born approximations are given with emphasis on the formulism used in the actual applications. Section 2 presents some numerical examples of the first and second Born approximations, and Section 3 discusses the impact of these results and future work.

1. THEORY

For an acoustic wave of frequency ω and sound speed c, the spatial part of the wave equation reduces to the Helmholtz equation

$$(\nabla^2 + k^2)P = 0 \tag{1}$$

where $k = \omega/c$, and P is the total pressure. The value of the field at some observation point in space due to the sound source and the field scattered from an adjacent surface can be obtained from Kirchhoff's Integral Theorem [8], i.e., for a sufficiently well-behaved function, the value of the field at the observation point in space can be expressed as an integral involving the function and its normal derivative evaluated on the surface enclosing the observation point. In practice, the enclosing surface of integration is taken as the scattering surface of interest plus a hemisphere at infinity. Integration over the hemisphere at infinity is assumed to give zero contribution. An additional small sphere encloses the source so that integration over the enclosed source does not produce a discontinuity; integration over this small sphere gives the contribution of the direct arrival from the source. Thus, only the integral over the scattering surface is required to obtain the scattered field.

In terms of the Green's function, G, the Helmholtz-Kirchhoff integral representation of the field is

$$P = P_i + \int_\Gamma (P_\Gamma \nabla G - G \nabla P_\Gamma) \cdot dS \tag{2}$$

where $\Gamma = \Gamma(x,y)$ is the surface from which the scattering occurs. The Green's function is defined by the equation

$$(\nabla^2 + k^2)G = \delta(\mathbf{R}) \tag{3}$$

P_i is the incident field, and P_Γ is the field at the scattering surface. We will assume in this paper (for illustrative purposes) that P_i is from a point source with a Gaussian "character," and is given by

$$P_i = D_o \exp[-(x^2/(2\sigma_x^2)) - (y^2/(2\sigma_y^2))]$$
$$\times \exp[i\mathbf{K_S}\cdot(\mathbf{R_S} + \mathbf{r})]/|\mathbf{R_S} + \mathbf{r}| \tag{4}$$

where $\mathbf{K_S}$ is the propagation vector from the source, σ_x^2 and σ_y^2 are the variances, and D_o is a constant.

The outgoing Green's function is

$$G = (1/4\pi)\ \exp[i\mathbf{K_R}\cdot(\mathbf{R_R} - \mathbf{r})]/|\mathbf{R_R} - \mathbf{r}| \tag{5}$$

and the surface scattering element is $\mathbf{dS} = \mathbf{n}\ dS$, where the unit normal $\mathbf{n}$ is

$$\mathbf{n} = \nabla(\xi - z)/|\nabla(\xi - z)|$$
$$= (\mathbf{i}\partial\xi/\partial x + \mathbf{j}\partial\xi/\partial y - \mathbf{k})/[(\partial\xi/\partial x)^2 + (\partial\xi/\partial y)^2 + 1]^{1/2} \tag{6}$$

In terms of the rough surface parameter, $\xi = \xi(x,y)$,

$$dS = [(\partial\xi/\partial x)^2 + (\partial\xi/\partial y)^2 + 1]\ dx\ dy \tag{7}$$

Thus, to what might be termed first order,

$$P_S^{(1)} = P^{(1)} - P_i = \int (P_i\nabla G - G\nabla P_i)\cdot\mathbf{dS} \tag{8}$$

where the field and its normal derivative at the surface have been assumed to be given by the familiar Kirchhoff approximations [2], $P_\Gamma = \mathcal{R}P_i$, and $\nabla P_\Gamma\cdot\mathbf{dS} = -\mathcal{R}\nabla P_i\cdot\mathbf{dS}$. Here, $\mathcal{R}$ is the plane wave reflection coefficient. Again, for illustrative purposes, $\mathcal{R}$ has been taken as unity (i.e., the surface chosen to illustrate the technique has $\mathcal{R} = 1$). In principle, the properties of the medium below the surface could be represented by this reflection coefficient.

When some assumption (i.e., some approximation) is made about the field at the surface, as has been done in Eq. (8), the resulting integral equation is sometimes commonly referred to as the Born approximation (if the integral is over a volume) or the Kirchhoff approximation (if the integral is over a surface) [2]. For historical reasons, we elect to refer to Eq. (8) as the first

order Born approximation although it is a surface integral. Eq. (8) may be written in the form,

$$P_S^{(1)} = \int P_i G \Gamma \cdot \mathbf{n} \, dx \, dy \tag{9}$$

where

$$\mathbf{n} = \mathbf{i}\partial\Gamma/\partial x + \mathbf{j}\partial\Gamma/\partial y - \mathbf{k} \tag{10}$$

$$\begin{aligned} \Gamma = {} & i(\mathbf{K_S} - \mathbf{K_R}) + (\mathbf{R_R} - \mathbf{r})/|\mathbf{R_R} - \mathbf{r}|^2 \\ & - (\mathbf{R_S} + \mathbf{r})/|\mathbf{R_S} + \mathbf{r}|^2 - ix/\sigma_x^2 - jy/\sigma_y^2 \end{aligned} \tag{11}$$

In actual applications, Eq. (9) is more conveniently written in the following form:

$$P_S^{(1)} = \alpha \int \beta\gamma \, dx \, dy \tag{12}$$

where

$$\alpha = D_o \exp[iK(R_S + R_R)]/4\pi R_S R_R \tag{13}$$

$$\begin{aligned} \beta = {} & \exp[-(x^2/(2\sigma_x^2)) - (y^2/(2\sigma_y^2))] \\ & \times \exp[iK(Ax + By - C\xi(x,y))]/[U(x,y)V(x,y)]^{1/2} \end{aligned} \tag{14}$$

$$\begin{aligned} \gamma = {} & [iKA + \{\sin\theta_R \sin\phi_R/(R_R U(x,y))\} \\ & + \{\sin\theta_S \sin\phi_S/(R_S V(x,y))\} - x/(R_R^2 U(x,y)) \\ & - x/(R_S^2 V(x,y)) - x/\sigma_x^2]\partial\xi/\partial x + [iKB \\ & + \{\sin\theta_R \cos\phi_R/(R_R U(x,y))\} \\ & + \{\sin\theta_S \cos\phi_S/(R_S V(x,y))\} - y/(R_R^2 U(x,y)) \\ & - y/(R_S^2 V(x,y)) - y/\sigma_y^2]\partial\xi/\partial y \\ & + [iKC + \{\cos\theta_R/(R_R U(x,y))\} + \{\cos\theta_S/(R_S V(x,y))\} \\ & + \xi/(R_R^2 U(x,y)) + \xi/(R_S^2 V(x,y))] \end{aligned} \tag{15}$$

and

$$A = \sin\theta_S \sin\phi_S - \sin\theta_R \sin\phi_R \tag{16}$$

$$B = \sin\theta_S \cos\phi_S - \sin\theta_R \cos\phi_R \tag{17}$$

$$C = \cos\theta_S + \cos\theta_R \tag{18}$$

A successive application of the above technique produces what we call the second order Born approximation:

$$P_S^{(2)} = \int_\Gamma (P_\Gamma^{(1)} \nabla_{r'} G - G \nabla_{r'} P_\Gamma^{(1)}) \cdot \mathbf{dS} \tag{19}$$

where

$$P_\Gamma^{(1)} = \int_\Gamma (P_\Gamma^{(0)} \nabla_r G_o - G_o \nabla_r P_\Gamma^{(0)}) \cdot \mathbf{dS} \tag{20}$$

$$G_o = (1/4\pi) \exp[i\mathbf{K}' \cdot (\mathbf{r}' - \mathbf{r})]/|\mathbf{r}' - \mathbf{r}| \tag{21}$$

$$G = (1/4\pi) \exp[i\mathbf{K_R} \cdot (\mathbf{R_R} - \mathbf{r}')]/|\mathbf{R_R} - \mathbf{r}'| \tag{22}$$

and $P_\Gamma^{(0)} = P_\Gamma^{(0)}(\mathbf{R_S}, \mathbf{r})$, where $\mathbf{r}$ is on the surface. $\mathbf{K}'$ is the reference wave vector that propagates the scattered field from the first scattering area increment, dS, to the second scattering area increment, dS'. In the case of deterministic surfaces, the applicable range of integration in the secondary scattering calculation can be found from a straightforward application of ray tracing. This has the added advantage of including some shadowing in the technique. (Depending upon the rough surface, shadowing can be an important and often neglected mechanism [3].)

Eq. (19) may be written as:

$$P_S^{(2)} = \int_\Gamma P_i^{(0)} G G_o \gamma' \Omega \cdot \mathbf{n}' \, dx' \, dy' \, dx \, dy \tag{23}$$

where

$$\gamma' = \Gamma' \cdot \mathbf{n}' \tag{24}$$

$$\Gamma' = i(\mathbf{K}' - \mathbf{K_S}) + (\mathbf{r}' - \mathbf{r})/|\mathbf{r}' - \mathbf{r}|^2 - (\mathbf{R_S} + \mathbf{r})/|\mathbf{R_S} + \mathbf{r}|^2 - ix/\sigma_x^2 - jy/\sigma_y^2 \tag{25}$$

and

$$\Omega = -i(\mathbf{K_R} + \mathbf{K}') + (\mathbf{R_R} - \mathbf{r}')/|\mathbf{R_R} - \mathbf{r}'|^2 + \mathbf{n}'/|\mathbf{r}' - \mathbf{r}|^2 + (\mathbf{r}' - \mathbf{r})/|\mathbf{r}' - \mathbf{r}|^2 \tag{26}$$

In actual applications, the second order Born approximation is more conveniently written in the following form:

$$P_S^{(2)} = \alpha' \int \beta' \gamma' \Omega \cdot \mathbf{n}' \, dx' \, dy' \, dx \, dy \tag{27}$$

where

$$\alpha' = D_o \exp[iK(R_S + R_R)]/((4\pi)^2 R_S R_R) \tag{28}$$

$$\beta' = \exp[-(x^2/(2\sigma_x^2)) - (y^2/(2\sigma_y^2))] \exp[iK\{A'x + B'y - C'\xi(x,y) + E'x' + F'y' + H'\xi(x',y')\}]/[U(x',y')V(x,y)R_o]^{1/2} \tag{29}$$

$$R_o^2 = (x' - x)^2 + (y' - y)^2 + (\xi(x',y') - \xi(x,y))^2 \tag{30}$$

$$A' = \sin\theta_S \sin\phi_S - x'/r' \tag{31}$$

$$B' = \sin\theta_S \cos\phi_S - y'/r' \tag{32}$$

$$C' = \cos\theta_S - \xi(x',y')/r' \tag{33}$$

$$E' = x'/r' - \sin\theta_R \sin\phi_R \tag{34}$$

$$F' = y'/r' - \sin\theta_R \cos\phi_R \tag{35}$$

$$H' = \xi(x',y')/r' - \cos\theta_R \tag{36}$$

$$\begin{aligned}\gamma' = {} & [iK(\sin\theta_S \sin\phi_S - (x' - x)/R_o) \\ & + \sin\theta_S \sin\phi_S/(R_S V(x,y)) + (x' - x)/R_o^2 \\ & - x/(R_S^2 V(x,y)) - x/(\sigma_x^2)]\partial\xi/\partial x \\ & + [iK(\sin\theta_S \cos\phi_S - (y' - y)/R_o) \\ & + \sin\theta_S \cos\phi_S/(R_S V(s,y)) + (y' - y)/R_o^2 \\ & - y/(R_S^2 V(x,y)) - y/(\sigma_x^2)]\partial\xi/\partial y \\ & + iK(\cos\theta_S - \xi/r) + \cos\theta_S/(R_S V(x,y)) \\ & - (z' - z)/R_o^2 + z/(R_S^2 V(x,y))\end{aligned} \tag{37}$$

$$\begin{aligned}V(x,y) = {} & (\sin\theta_S \sin\phi_S - x/R_S)^2 + (\sin\theta_S \cos\phi_S - y/R_S)^2 \\ & + (\cos\theta_S - \xi/R_S)^2\end{aligned} \tag{38}$$

$$\begin{aligned}\Omega\cdot\mathbf{n}' = {} & [-iK(\sin\theta_R \sin\phi_R + (x' - x)/R_o) \\ & + \sin\theta_R \sin\phi_R/(R_R U(x',y')) \\ & + (x' - x)/R_o - x'/(R_R^2 U(x',y'))]\partial\xi(x',y')/\partial x' \\ & + [-iK(\sin\theta_R \cos\phi_R + (y' - y)/R_o) \\ & + \sin\theta_R \cos\phi_R/(R_R U(x',y')) \\ & + (y' - y)/R_o^2 = y'/(R_R^2 U(x',y'))]\partial\xi(x',y')/\partial y' \\ & + iK(\cos\theta_R - (\xi(x',y') - \xi(x,y))/R_o^2) \\ & - \cos\theta_R/(R_R U(x',y')) + (\xi(x',y') \\ & + (\xi(x',y') - \xi(x,y))/R_o^2\end{aligned} \tag{39}$$

and

$$\begin{aligned}U(x',y') = {} & (\sin\theta_R \sin\phi_R - x'/R_R)^2 \\ & + (\sin\theta_R \cos\phi_R - y'/R_R)^2 \\ & + (\cos\theta_R - \xi(x',y')/R_R)^2\end{aligned} \tag{40}$$

The second order Born approximation, as outlined above, can be used to account for a second scattering event from an adjacent area of the rough surface. This procedure can be continued to obtain even higher orders of multiple scattering (i.e., a third scattering event, fourth scattering event, etc.). In general

$$P_S = P_S^{(1)} + P_S^{(2)} + \cdots + P_S^{(N)} + \cdots \tag{41}$$

where

$$P_S^{(N)} = \int (P_S^{(N-1)} \nabla G - G \nabla P_S^{(N-1)}) \cdot d\mathbf{S} \tag{42}$$

and G is the appropriate Green's function. However, in actual applications where it is desirable to include the effects of multiple scattering, the second order Born approximation has been found to be sufficient, i.e., further higher orders are usually not needed.

2. NUMERICAL EXAMPLES

A numerical example was selected to illustrate the use of the second order Born approximation and its ability to account for secondary scatter. Therefore, in the example that follows, no attempt was made to rigorously enforce the Kirchhoff approximation criterion for deterministic surfaces [9,10], namely,

$$2k|R_c|\sin^3\theta_g >> 1 \tag{43}$$

where R_c is the radius of curvature of the surface, θ_g is the local grazing angle of the incident field, and k is the acoustic wavenumber.

A large scale rough surface was simulated by a rigid (hard) sinusoidally corrugated surface given by the equation $\xi = (A)\sin(2\pi x/\lambda)$, where A and λ are surface parameters (in meters). This rough surface was ensonified by a low frequency (100-200 Hz) Gaussian-like beam with variances $\sigma_x = 2$, $\sigma_y = 2$, and $D_o = 100$. The distance from the source to the rough ensonified area was 300 meters and the distance from the rough ensonified area to the receiver was also 300 meters. The angle made by the incident field and the reference scattering surface was 30°. Both the source and receiver were assumed to be in water with a sound speed of 1500 meters/second.

Although the methodology developed here allows for any piecewise continuous surface, it is sufficient here to restrict

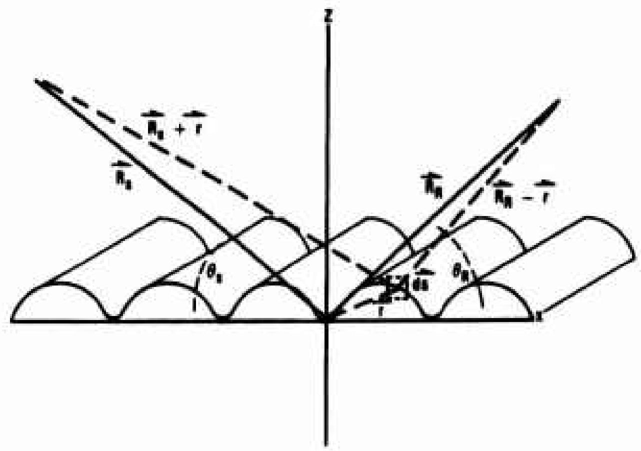

Fig. 1. Schematic showing the coordinate system and position vectors for the first order Born approximation.

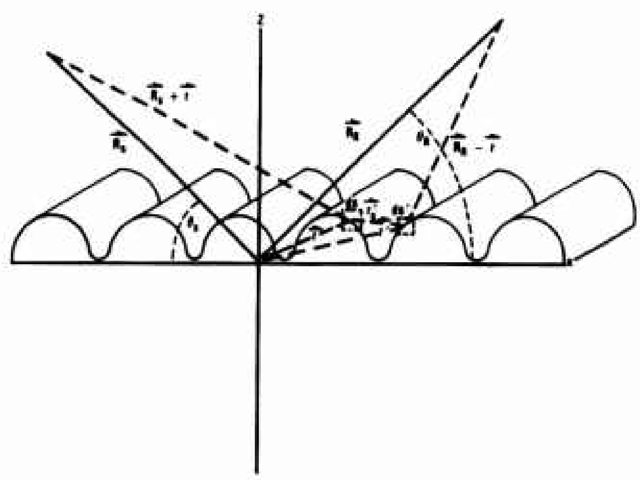

Fig. 2. Schematic showing the coordinate system and position vectors for the second order Born approximation.

calculations to a corrugated sinusoidal surface. The effect of shadowing and penetration will be examined in a subsequent work, the emphasis here being on secondary scattering from the surface (i.e., second Born contributions and its comparison to first order contributions). One of the most useful ways to investigate the influence of second Born is to present calculations of an incident

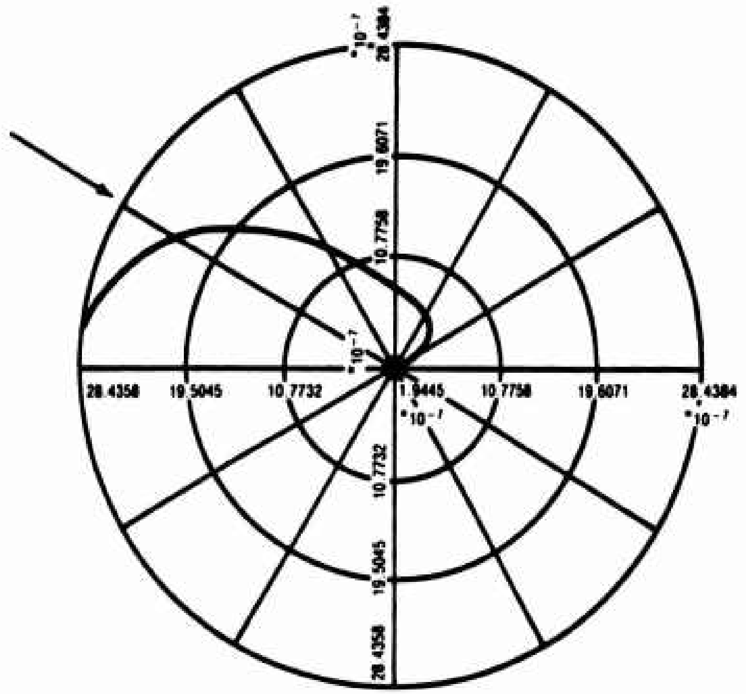

Fig. 3. Polar plot showing the results of the first order Born approximation for a 100 Hz Gaussian beam incident at a grazing angle of 30° (indicated by arrow) and scattered from a rigid sinusoidally corrugated surface of parameters A = 2 m and λ = 4 m.

field and plot the scattered field from 0° to 180°. We will present results for different surface roughness amplitudes (A) and displacements (λ) from the peak amplitudes. The frequencies for the incident field were 100 Hz and 200 Hz. Later we will present results as the frequency varies from 0 to 1000 Hz. We do not make claims on the validity of the method in the lower frequency domain; we simply wish to examine the first two contributions. In principle, inclusion of all order terms should converge to the exact answer using this method.

Figure 3 illustrates the case for scattering from a surface with λ = 4 m and A = 2 m for a frequency of 100 Hz. The region of integration for $\sigma_x = \sigma_y = 2$ is 16 by 16 square meters. This guarantees that the integrand has died off sufficiently (due to Gaussian spreading) to have included an adequate area. The method used to perform the double integration on the surface was Gauss-Legendre integration with a suitable number of weighting terms to ensure convergence. Since the surface amplitude A is fairly large for this frequency, the most significant return is in the backward direction. The maximum response is at approximately 8° in the

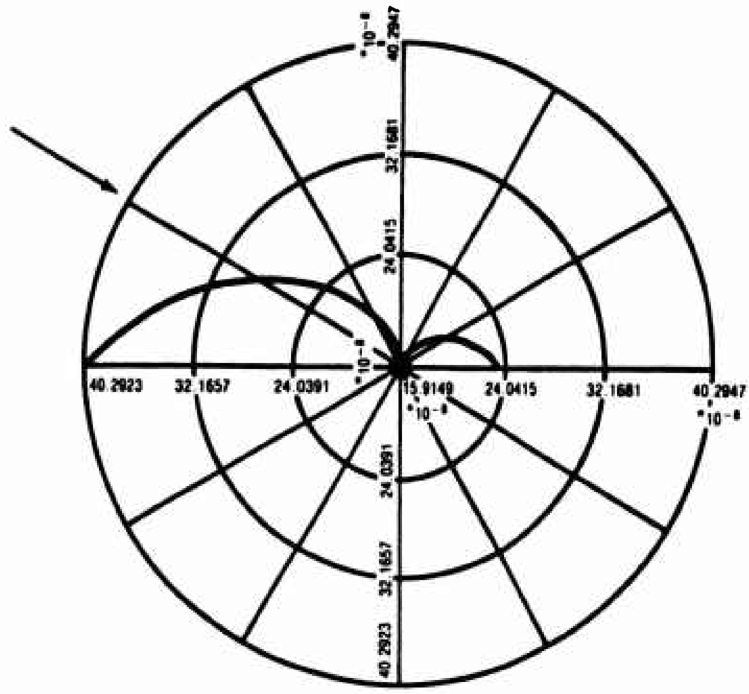

Fig. 4. Polar plot showing the results of the second order Born approximation for a 100 Hz Gaussian beam incident at a grazing angle of 30° and scattered from a rigid sinusoidally corrugated surface of parameters A = 2m and λ = 4m.

backward direction with a value of 2.8×10^{-6} (arbitrary units). Figure 4 illustrates the case for the second Born contribution. The maximum contribution here is down by almost a factor of ten over the first order term. The contribution is primarily in the backward direction with a significant component in the forward direction and little contribution in the region normal to the surface. In fact, this behavior was fairly consistent for all surfaces examined in this study for second Born while rather pronounced differences were absent for first order contributions. One may be tempted to assume that because second order was about an order of magnitude smaller than first order, it had little influence on the total results. That, in fact, is not the case since the two quantities add coherently leading to a large overall effect as can be seen in Fig. 5. In particular, the backward scattering is increased by 15% and in the most forward direction, by over 100%. (Note that the scales are different in Figs. 3, 4, and 5.)

We now examine an example of a surface of smaller amplitude (A = 0.5 m) in which the spacing of the sinusoidal peaks are more

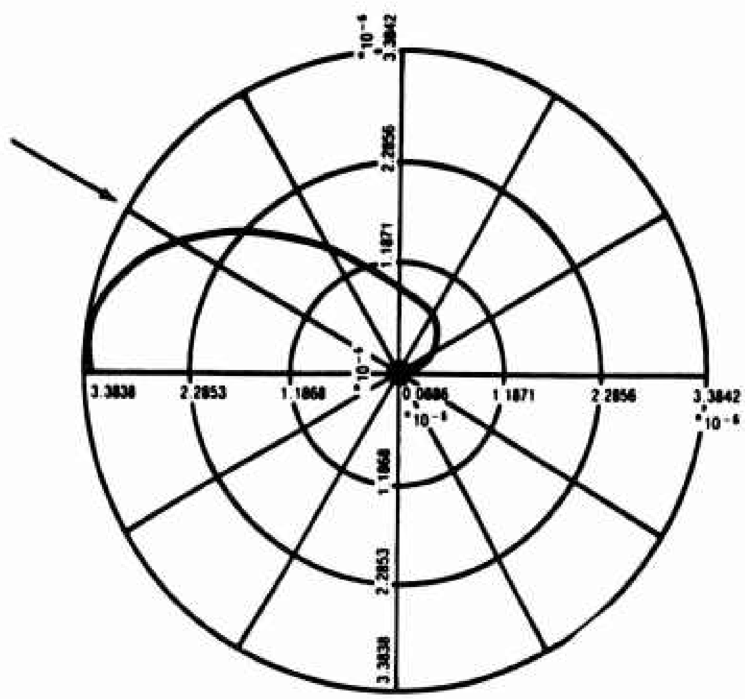

Fig. 5. Polar plot showing the combined results of the first and second order Born approximations for a 100 Hz Gaussian beam incident at a grazing angle of 30° and scattered from a rigid sinusoidally corrugated surface of parameters A = 2m and λ = 4m.

closely spaced. As in the earlier case, the incident field is at 30° (indicated by the arrow in the figures) and at 100 Hz. Now, however, as shown in Fig. 6, the total response (solid curve) and the first order response (dotted curve) are focused in the forward direction at an angle slightly above 60° (with respect to the horizontal). The second order contribution (dashed curve) appears fairly omnidirectional on this scale although it is actually quide similar in detail to Fig. 4. It is clear that the second order contribution is quite substantial for this case especially in the backward direction. Indeed, without the second order component, the backscattered response would be about a factor of 40 too small. Even in the forward direction, where first order leads to its largest response, introduction of the second order term enhances the total contribution by over 70%. It is therefore not surprising to find that the Kirchhoff approximation is noted to yield inadequate results in the backward direction for cases such as this.

Figure 7 shows the result of scattering from the same rough interface as in the example just discussed except at a higher

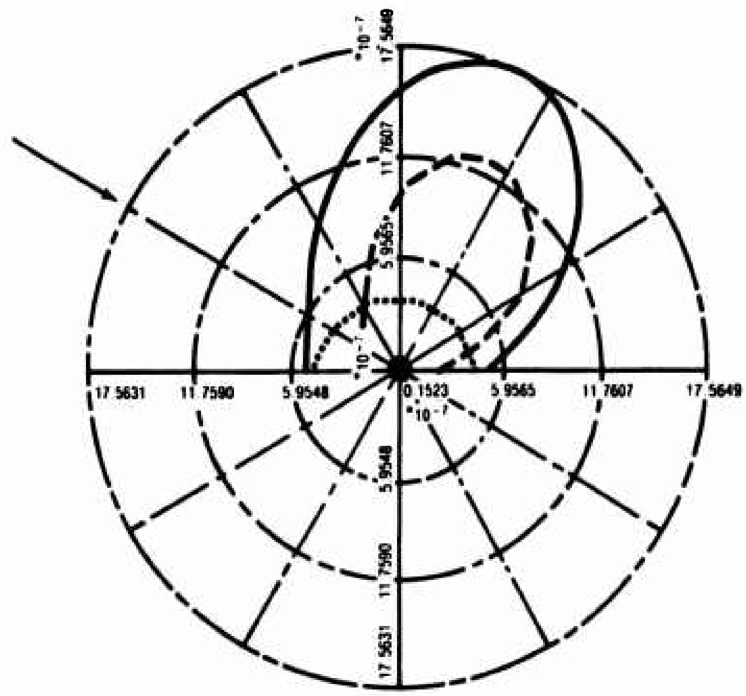

Fig. 6. Polar plot showing the results of the first, second, and combined Born approximations for a 100 Hz Gaussian beam incident at a grazing angle of 30° and scattered from a rigid sinusoidally corrugated surface of parameters A = 0.5 m and λ = 2 m.

frequency. The frequency now is at 200 Hz and it is clear from the figure that the second order contribution is not as significant as in the lower frequency case. In this case the incident field is again 30° relative to the horizontal and the scattered field is just below 60° (relative to the horizontal) in the forward direction. Note that the width of the scattered response (lobe) is narrower than the 100 Hz case as one would expect as higher frequencies are approached. From the last two examples it might be expected that the second order contribution decreases in importance relative to first order with increasing frequency. However, for particular angles and surfaces, there are situations for which second order effects are very important. Such situations can happen when first order contributions are not allowed to escape the surface but scatter back into the surface once and then proceed outward. This could happen at higher frequencies but it is not likely at lower frequencies.

Figures 8, 9, and 10 show first order, second order, and total scattering, respectively, from a corrugated surface as

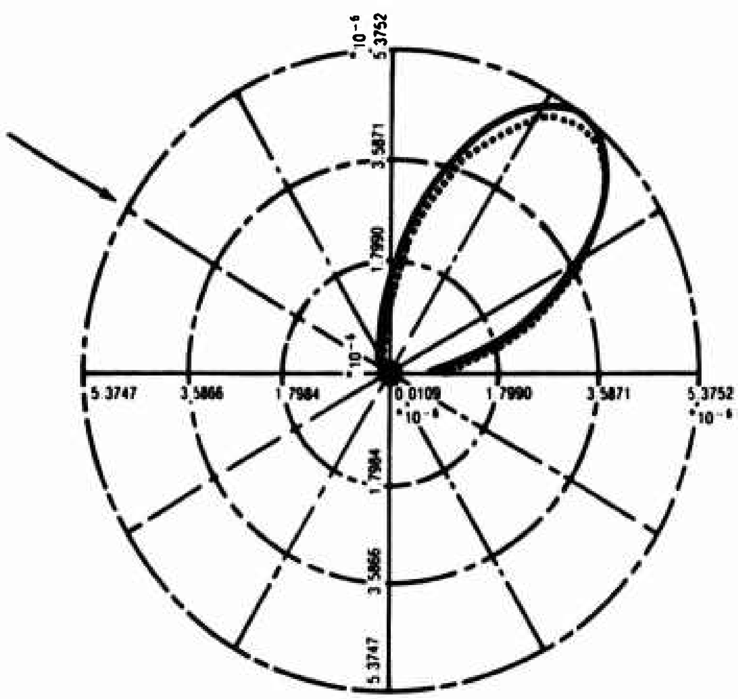

Fig. 7. Polar plot showing the results of the first, second, and combined Born approximations for a 200 Hz Gaussian beam incident at a grazing angle of 30° and scattered from a rigid sinusoidally corrugated surface of parameters A = 0.5 m and λ = 2 m.

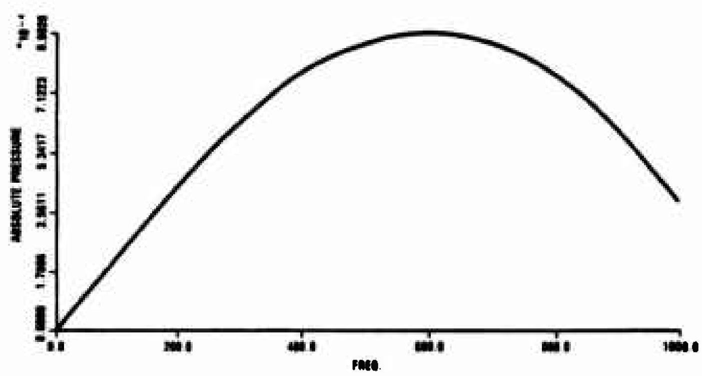

Fig. 8. Plot of pressure calculated using the first order Born versus frequency for a rigid sinusoidally corrugated surface.

frequency varies from 0 to 1000 Hz. Since the surface is periodic the first order term approaches a Bragg pattern. The second order contribution varies moderately from about 20 Hz to 400 Hz and then begins to rise in value, though not smoothly. (The incident angle is 30° from the horizontal, and the angle of detection is 30° in the forward direction, also measured from the horizontal.)

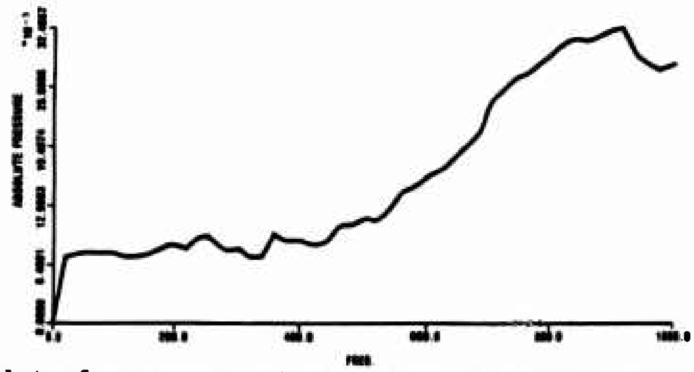

Fig. 9. Plot of pressure calculated using second order Born versus frequency for a rigid sinusoidally corrugated surface.

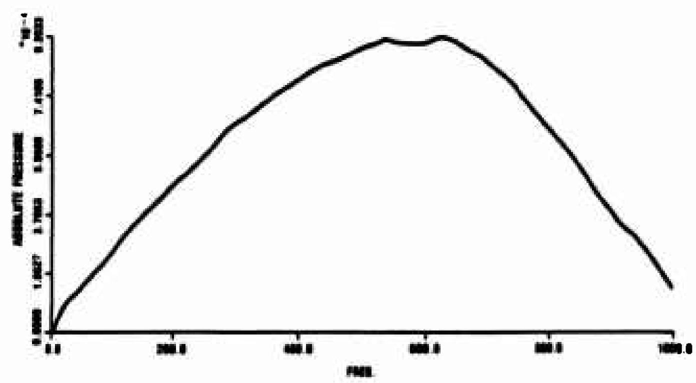

Fig. 10. Plot of composite pressure calculated using first and second order Born versus frequency for a rigid sinusoidally corrugated surface.

Figure 10 illustrates the coherent sum of the two contributions. What is interesting is that the two add constructively out to 600 Hz and destructively beyond 600 Hz where second Born begins to dominate due to the Bragg scattering effect on the first order term. In general, we might expect that second order effects "wash out" the nice Bragg nulls observed in the first order calculations; but, due to the destructive interference effect in the frequency region where second Born becomes important, a pronounced dip is still observed. This is probably peculiar to this example. In general, it is expected that second order will alter the periodic Bragg pattern.

3. SUMMARY

A technique, the "second order Born approximation," has been presented which can account for multiple scattering and some shadowing. Scattering integrals are numerically integrated directly,

thereby avoiding many of the approximations used in other approaches to scattering. The technique is equally suitable for near-field and far-field calculations. An illustrative example has been presented which shows that in some cases where large scale rough surface scattering occurs, secondary scattering can be significant; and failure to include secondary scattering can lead to erroneous results.

The examples presented were limited due to the large expense in computer costs in including second Born and because the neglect of including shadowing precluded consideration of lower grazing angles. Future upgrades of the model will enable faster calculations that will include both shadowing and penetration. Clearly, a more comprehensive numerical study including the higher frequency range will be of considerable interest.

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